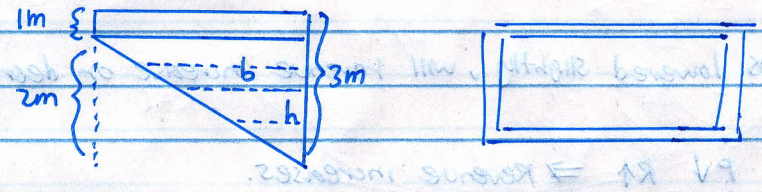
Related Rate

Steps :

① Read the problem carefully, sketch & organize given information
② Identify the rates given and rates to be determined.
③ Write down the equations that express the basic relationships among the variables.
④ Substitute known values and solve for the desired quantity.
⑤ Check for consistancy of solution (eg. "sign")

- Eg) A swimming pool is 50 meters long and 20 meters wide. The depth decreases linearly along the length from 3m to 1m. It is initially empty and filled at a rate of $1\frac{m^3}{hr}$. How fast is the water level rising 250min after the filling begins? How long will it take to fill the pool?



Ans : By similar triangles.

$$\frac{h}{b} = \frac{2}{50} \Rightarrow \frac{h}{b} = \frac{1}{25} \Rightarrow b = 25h$$

$$A = \frac{1}{2}bh = \frac{1}{2} \cdot 25h \times h = 12.5h^2$$

Volume $V = V(h) = A \times \text{width} = A \times 20 = 12.5h^2 \times 20 = 250h$

$$\frac{dV}{dt} = \frac{dV}{dh} \cdot \frac{dh}{dt} = 500h \cdot \frac{dh}{dt} = 500 \cdot 1 \cdot \frac{dh}{dt} = 1 m^3/hr$$

$$\Rightarrow \frac{dh}{dt} = 1 m^3/hr / 500$$

For $2 < h \leq 3$, $V(h) = 250 \cdot 2^2 + 50 \cdot 20 \cdot h \Rightarrow V = 1000 + 1000h \Rightarrow V(h) = 1000 + 1000h$

When $h = 1$, $\frac{dV}{dt} = 500 \cdot 1 \cdot \frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{1}{500}$ m/min $\Rightarrow 0.002$ m/min

$\Rightarrow 2$mm/min

Volume of the pool $= 2000 m^2$, So at $1 m^3/hr$, it will take 200 min.